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Lattice monopole action in pure SU(3) QCD

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Abstract: We obtain an almost perfect monopole action numerically after abelian projection in pure SU(3) lattice QCD. Performing block-spin transformations on the dual lattice, the action fixed depends only on a physical scale b. Monopole condensation occurs for large b region. The numerical results show that two-point monopole interactions are dominant for large b. We next perform the block-spin transformation analytically in a simplified case of two-point monopole interactions with a Wilson loop on the fine lattice. The perfect operator evaluating the static quark potential on the coarse b-lattice are derived. The monopole partition function can be transformed into that of the string model. The static potential and the string tension are estimated in the string model framework. The rotational invariance of the static potential is recovered, but the string tension is a little larger than the physical one.

Keywords: Solitons Monopoles and Instantons, Confinement, Lattice Gauge Field Theories.

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Contents

1. Introduction

The quark confinement is a key problem to understand non-perturbative phenomena of QCD. The dual Meissner effect is a promising candidate [1]. 't Hooft suggested the idea of abelian projection [2]. SU(N) QCD is reduced to a U(1)$^{N-1}$ abelian gauge theory with magnetic monopoles by a partial gauge fixing (abelian projection). The dual Meissner effect is caused by the monopole condensation. Monopoles are responsible for the confinement as in compact QED [3]–[8].

Monte-Carlo simulations of lattice QCD are a most powerful method to study the non-perturbative phenomena. Numerical studies of the abelian projection in the maximally abelian (MA) gauge have confirmed the 't Hooft conjecture. In MA gauge [9, 10], the string tension derived from abelian Wilson loops gives almost the same value as that of non-abelian Wilson loop (abelian dominance) [12]–[13]. Moreover, the monopole contribution to the abelian Wilson loops alone reproduces the string tension in SU(2) QCD (monopole dominance) [14]–[16]. The abelian and the monopole dominances are seen also in the behavior of the Polyakov loop in $T \neq 0$ SU(2) and SU(3) QCD [17]–[19]. These results support the 't Hooft conjecture.

Wilson's idea of a block-spin transformation and a renormalized trajectory (RT) are useful when we study the continuum limit on available lattices [20]. The lattice action on RT has no lattice artifact and hence it is called quantum perfect action. It reproduces the same physical results as in the continuum limit. It is challenging to get the perfect lattice action for the infrared region of QCD. For that purpose, we have to extract a dynamical variable which plays a dominant role in the infrared region. The numerical evidence for the monopole dominance suggests that low-energy QCD can be described by an effective action on the dual lattice in terms of a dual quantity like monopole currents. It is very interesting to obtain such an effective monopole action. Also it is challenging to set the perfect monopole action with the help of block-spin transformations for monopole currents. The monopole action can be obtained numerically by an inverse Monte-Carlo method [21]. A block-spin transformation on the dual lattice can be performed by considering an n-blocked monopole current [22]. The detailed study of the effective monopole action in pure SU(2) QCD has been done [23]–[25]. The monopole action determined in SU(2) has the following important features:

1. Two-point interactions are dominant in the infrared region and coupling constants decrease rapidly as the distance between two monopole currents increases.

2. The action fixed seems to satisfy the scaling behavior, that is, it depends only on a physical scale $b = na(\beta)$. This suggests that the action is near to RT.

3. Monopole condensation seems to occur for large b region from the energy-entropy balance.

In order to test the validity of the statement that the action is near to RT, we need to check the restoration of the continuum rotational invariance. Then, we have to determine first the correct forms of physical operators (quantum perfect operators) on the blocked lattice. In the above pure SU(2) study, S. Fujimoto et al. [26] have taken the following steps:

1. Study the renormalization flow on the projected space of two-point monopole interactions alone, considering that they are dominant numerically in the infrared region.

2. The static potential between quark and antiquark can be evaluated by the expectation value of the Wilson loop in the continuum limit. Hence the Wilson loop can be regarded as the correct operator evaluating the static potential on the fine a-lattice also.

3. Perform the block spin transformation *analytically*, starting from the two-point monopole interactions with the Wilson loop on the fine a-lattice. The quantum perfect action and the quantum perfect operator on the coarse b-lattice can be obtained *analytically* when we take the $a \to 0$ limit for fixed $b = na$.

4. Compare the quantum perfect action composed of two-point monopole interactions with the effective action numerically determined from the inverse Monte-Carlo method. The parameters in the perfect monopole action and the quantum perfect operator are fixed then.

5. The effective action with the quantum perfect operator can be transformed into that of the string model. Since the strong-coupling expansion is found to work well in the string model, we see that the static potential is estimated analytically by the classical part alone and the continuum rotational invariance is restored.

It is not so straightforward to extend these studies to pure SU(3) QCD, since there are three monopole currents $k_\mu^{(a)}(s)$ satisfying one constraint $\sum_{a=1}^{3} k_\mu^{(a)}(s) = 0$. So far, SU(3) monopole action composed of only one kind of monopole current after integrating out the other two has been derived in the case of two-point interactions [27]. In this case, the same method can be applied as done in pure SU(2) QCD and monopole condensation seems to occur from the energy-entropy balance in rather strong coupling region. But the scaling seen in the pure SU(2) case was not seen. Also it is important to study effective action for two independent monopole currents in order to see the characteristic features of pure SU(3) QCD.

The purpose of this paper is to report the results of the extensive studies of SU(3) monopole actions, especially in terms of two independent monopole currents. In section 2 we briefly review the SU(3) monopole current on the lattice and the inverse Monte-Carlo method. The numerical results of the SU(3) monopole action are shown in section 3. In section 4 we perform a block-spin transformation of the monopole current analytically, considering only the infrared dominant two-point monopole interactions. In section 5, transforming the monopole action into that of the string model, we calculate the static potential analytically. In section 6 the string tension is estimated from the results of previous sections. The conclusions are given in section 7. The correspondence between the monopole action and the dual abelian Higgs theory (dual Ginzburg-Landau theory) is given in appendix.

2. Monopole current and the inverse Monte-Carlo method

We extract a $U(1)^2$ link field $u_\mu(s)$ from a SU(3) link field $U_\mu(s)$ after abelian projection called MA gauge. In the SU(2) case, a U(1) link field is defined as $u_\mu(s) = \mathrm{diag}(\mathrm{e}^{\mathrm{i}\theta_\mu^{(1)}}, \mathrm{e}^{\mathrm{i}\theta_\mu^{(2)}})$, where $\theta_\mu^{(a)}(s) \equiv \arg[U_\mu(s)]_{aa}$ $(a = 1, 2)$. It satisfies $\det(u_\mu(s)) = 1$ due to $\sum_{a=1}^{2} \theta_\mu^{(a)} = 0$. Since $\sum_{a=1}^{3} \theta_\mu^{(a)} \neq 0$ in the SU(3) case, the definition of $U(1)^2$ link field necessarily becomes more complicated. In this study, we use the definition of ref. [28, 29]. footnote Another definition is seen in ref. [10]. Both definitions are equivalent in the continuum limit. The fields transforming as photon fields under

$U(1)^2$ are defined as follows:

$$\begin{aligned}\theta_\mu^{(a)} &\equiv \arg[U_\mu]_{aa} - \frac{1}{3}\phi_\mu\,,\\ \phi_\mu &\equiv \sum_{a=1}^{3}\arg[U_\mu]_{aa}\Big|_{\mathrm{mod}\ 2\pi} \in [-\pi,\pi)\,.\end{aligned}\tag{2.1}$$

The $U(1)^2$ link field defined by $u_\mu(s) = \mathrm{diag}(e^{i\theta_\mu^{(1)}}, e^{i\theta_\mu^{(2)}}, e^{i\theta_\mu^{(3)}})$ satisfies $\det(u_\mu(s)) = 1$.

If the $U(1)^2$ field strength is defined as $\overline{\Theta}_{\mu\nu}^{(a)} \equiv \partial_\mu\theta_\nu^{(a)} - \partial_\nu\theta_\mu^{(a)} \pmod{2\pi}$, then $\sum_a \overline{\Theta}_{\mu\nu}^{(a)} = 2\pi l$ ($l = 0, \pm 1$) and is not always zero. When $l = +1(-1)$, $\overline{\Theta}_{\mu\nu}^{(a)}$ is redefined. If $\overline{\Theta}_{\mu\nu}^{(a)}$ is the maximum(minimum) of $(\overline{\Theta}_{\mu\nu}^{(1)}, \overline{\Theta}_{\mu\nu}^{(2)}, \overline{\Theta}_{\mu\nu}^{(3)})$, it is redefined as $\overline{\Theta}_{\mu\nu}^{(a)} - 2\pi(+2\pi)$. Others do not change. Then new $U(1)^2$ field strengths satisfy $\sum_a \overline{\Theta}_{\mu\nu}^{(a)} = 0$. Monopole currents are defined as $k_\mu^{(a)} \equiv 1/4\pi\epsilon_{\mu\nu\rho\sigma}\partial_\nu\overline{\Theta}_{\rho\sigma}^{(a)}$ by DeGrand-Toussaint(D-T) [7] which satisfies the constraint

$$\sum_a k_\mu^{(a)} = 0\,.\tag{2.2}$$

We want to get an effective monopole action on the dual lattice integrating out the degrees of freedom except for the monopole currents:

$$\begin{aligned}Z &= \int \mathcal{D}U\delta(X^{off})\Delta_F(U)e^{-S(U)}\\ &= \int \mathcal{D}u\left[\int \mathcal{D}c\delta(X^{off})\Delta_F(U)e^{-S(U)}\right]\\ &= \int \mathcal{D}ue^{-S_{\mathrm{eff}}(u)}\\ &= \sum_{k^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu k_\mu^{(a)},0}\delta_{\Sigma_a k^{(a)},0}\int \mathcal{D}u\delta(k^{(a)},u)e^{-S_{\mathrm{eff}}(u)}\\ &= \sum_{k^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu k_\mu^{(a)},0}\delta_{\Sigma_a k^{(a)},0}e^{-S[k^{(a)}]}\,,\end{aligned}\tag{2.3}$$

where $U_\mu(s) = c_\mu(s)u_\mu(s)$ and X^{off} are the off-diagonal part of the following quantity:

$$X(s) = \sum_{\mu,a}[U_\mu(s)\Lambda_a U_\mu^\dagger(s) + U_\mu^\dagger(s-\hat\mu)\Lambda_a U_\mu(s-\hat\mu), \Lambda_a]\,,\tag{2.4}$$

$$\Lambda_1 = \begin{pmatrix}1&0&0\\0&-1&0\\0&0&0\end{pmatrix},\qquad \Lambda_2 = \begin{pmatrix}-1&0&0\\0&0&0\\0&0&1\end{pmatrix},\qquad \Lambda_3 = \begin{pmatrix}0&0&0\\0&1&0\\0&0&-1\end{pmatrix}.\tag{2.5}$$

$\Delta_F(U)$ is the Faddeev-Popov determinant and $\delta(k^{(a)},u)$ is the delta function corresponding to the D-T definition of the monopole current.

We can perform the above integration numerically. We create vacuum ensembles of monopole currents $\{k_\mu^{(a)}(s)\}$ using the Monte-Carlo method and the above definition of the monopole current. Then we derive the effective monopole action using the Swendsen method [21] which is one of the inverse Monte-Carlo methods.

In order to take the continuum limit, we perform the block-spin transformation on the dual lattice by defining an n-blocked monopole current [22]:

$$K_\mu^{(a)}(s^{(n)}) \equiv \sum_{i,j,m=0}^{n-1} k_\mu^{(a)}\left(ns + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + m\hat{\sigma}\right). \tag{2.6}$$

If the action numerically obtained satisfies a scaling behavior $S[K^{(a)}, n, a(\beta)] = S[K^{(a)}, b = na(\beta)]$, that is, the action depends only on b, the continuum limit can be taken as $n \to \infty, a(\beta) \to 0$ for a fixed physical length $b = na(\beta)$.

The original Swendsen method must be extended due to the conservation law of the monopole current $\partial'_\mu k_\mu^{(a)}(s) = 0$ as in the SU(2) case [23]. Let us assume the form of the monopole action as $S[k^{(a)}] = \sum_i G_i S_i[k^{(a)}]$ (G_i is the coupling constant for the operator $S_i[k^{(a)}]$) and define $\tilde{S}_i[k^{(a)}]$ as a part of $S_i[k^{(a)}]$ which contains the currents around a specific plaquette $(s', \hat{\mu}', \hat{\nu}')$. When we consider the expectation value of some operator $O_i[k^{(a)}]$, the following identity holds as in the SU(2) case:

$$\langle O_i[k^{(a)}]\rangle = \langle \overline{O}_i[k^{(a)}]\rangle\,, \tag{2.7}$$

$$\langle O_i[k^{(a)}]\rangle \equiv \frac{\sum_{k^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu k_\mu^{(a)},0}\delta_{\Sigma_a k^{(a)},0} O_i[k^{(a)}] \exp(-\sum_j G_j S_j[k^{(a)}])}{\sum_{k^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu k_\mu^{(a)},0}\delta_{\Sigma_a k^{(a)},0} \exp(-\sum_j G_j S_j[k^{(a)}])}\,, \tag{2.8}$$

$$\overline{O}_i[k^{(a)}] \equiv \frac{\sum_{M^{(a)}\in\mathbb{Z}} \delta_{\Sigma_a M^{(a)},0} O_i[k'^{(a)}] \exp(-\sum_j G_j \tilde{S}_j[k'^{(a)}])}{\sum_{M^{(a)}\in\mathbb{Z}} \delta_{\Sigma_a M^{(a)},0} \exp(-\sum_j G_j \tilde{S}_j[k'^{(a)}])}\,, \tag{2.9}$$

$$k'^{(a)}_\mu(s) \equiv k_\mu^{(a)}(s) + M^{(a)}(\delta_{s,s'}\delta_{\mu,\mu'} + \delta_{s,s'+\hat{\mu}'}\delta_{\mu,\nu'} - \delta_{s,s'+\hat{\nu}'}\delta_{\mu,\mu'} - \delta_{s,s'}\delta_{\mu,\nu'})\,. \tag{2.10}$$

We use this identity to determine the values of the couplings G_i iteratively. Choose trial couplings $\tilde{G}_i$ suitably. If $\tilde{G}_i$ are not equal to G_i for all i, then $\Delta\tilde{G}_i \equiv G_i - \tilde{G}_i$ are estimated from the expansion of eq. (2.7) up to the first order of $\Delta\tilde{G}_i$:

$$\langle O_i - \overline{O}_i\rangle \cong \langle \overline{O}_i\overline{S}_j - \overline{O_i S_j}\rangle \Delta\tilde{G}_j\,. \tag{2.11}$$

$\tilde{G'}_i = \tilde{G}_i + \Delta\tilde{G}_i$ are used as the next trial couplings. These procedures continue iteratively until the couplings converge.

3. Numerical results

Practically, we must truncate the form of the monopole action to derive it numerically. We know that short-distant and two-point interactions are dominant in the

SU(2) case. Here, we assume the following form of the SU(3) monopole action:

$$\begin{aligned} S[k^{(a)}] &= \sum_i G_i \left(S_i[k^{(1)}] + S_i[k^{(2)}] + S_i[k^{(3)}]\right) \\ &= \sum_i G_i \left(S_i[k^{(1)}] + S_i[k^{(2)}] + S_i[-k^{(1)} - k^{(2)}]\right). \end{aligned} \tag{3.1}$$

The Weyl symmetry (the species permutation symmetry of the monopole currents) remains after abelian projection. We adopt 27 two-point interactions whose distances are up to $3na(\beta)$ [23] and 4, 6-point interactions of the following simple form:

$$\sum_{a,s}\left\{\sum_{\mu=-4}^{4}(k_\mu^{(a)}(s))^2\right\}^2, \qquad \sum_{a,s}\left\{\sum_{\mu=-4}^{4}(k_\mu^{(a)}(s))^2\right\}^3. \tag{3.2}$$

The lattice size is 48^4 from $\beta = 5.6$ to $\beta = 6.4$. After thermalization, 30 configurations are used for the average. We determine the lattice spacing $a(\beta)$ without using the theoretical asymptotic beta function. It is given by the relation $a(\beta) = \sqrt{\sigma(\beta)/\sigma_{ph}}$ [30], where σ_{ph} is the physical string tension.[1] The results are summarized as follows:

1. The monopole action for two independent types of monopole currents is obtained clearly (figure 1). The qualitative behaviors are the same as in the SU(2) monopole action:

   (a) The monopole action has a compact form. The self-interaction $G_1 \sum_{a,s,\mu}(k_\mu^{(a)}(s))^2$ is dominant and the coupling constants G_i decrease rapidly as the distance between the two monopole currents increases.

   (b) The coupling constants have a direction dependence which is expected after blocking. Two nearest-neighbor interactions $G_2 \sum_{a,s,\mu} k_\mu^{(a)}(s)k_\mu^{(a)}(s+\hat{\mu})$ and $G_3 \sum_{a,s,\mu\neq\nu} k_\mu^{(a)}(s)k_\mu^{(a)}(s+\hat{\nu})$ are quite different for small b region.

   (c) The coupling constants G_i become very small for large b region.

   (d) The simple 4, 6-point interactions become negligibly small for large $b > 2\sigma_{ph}^{-1/2}$. Two-point interactions are relatively dominant for large b region.

   (e) The scaling behavior holds well for $n = 4, 6, 8$ data, if the physical scale $b = na(\beta)$ is taken in unit of the string tension $\sqrt{\sigma_{ph}}$. The action seems to be very near to RT on which one can take the continuum limit.

2. In order to study if monopole condensation occurs by energy-entropy balance, we derive the monopole action, considering only one type of the monopole

[1] If we use $\sigma_{ph}^{1/2} = 440\,\mathrm{MeV}$, $b = 2\sigma_{ph}^{-1/2}$ is approximately equal to $0.9\,\mathrm{fm}$.

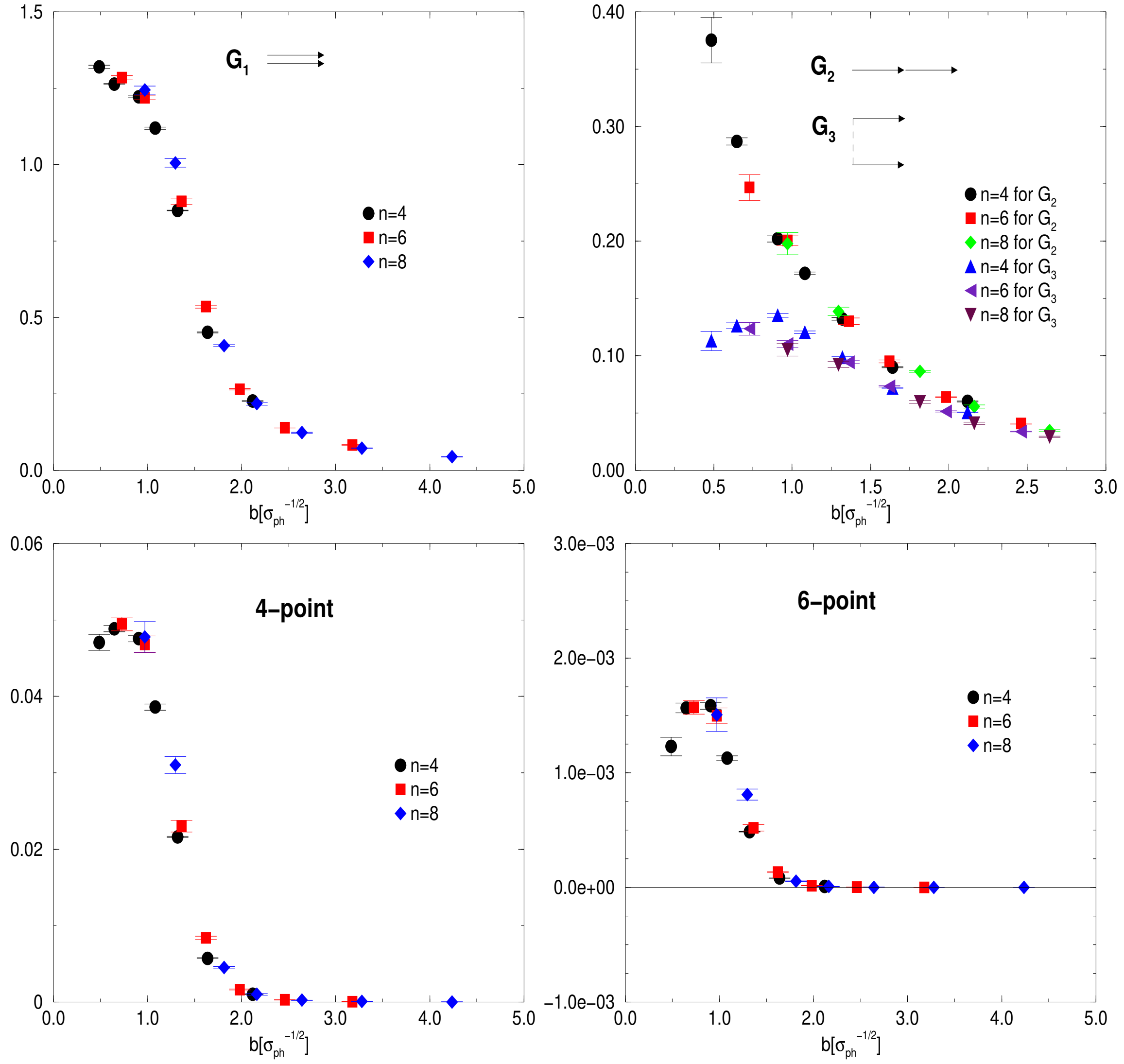

Figure 1: The coupling G_i versus b in two-current case. The 4pt couplings are negative really.

JHEP02(2000)012

current. For simplicity, only two-point interactions are taken into account. The scaling was not seen in the previous study where the two loop perturbation value was used for $a(\beta)$ [27]. When $a(\beta)$ is fixed by the string tension, the scaling is seen beautifully in figure 2. Since we restrict ourselves to one type of the monopole current, the entropy of the monopole current loop is given approximately by $\ln 7 \times L$ (L is a length of one long loop). From the previous study, we know that only one long loop and some short loops of monopoles exist in the vacuum and the value of the action is well approximated as $G_1 \times L$. Figure 2 shows that the entropy dominates over the energy in the large b region, i.e. $G_1(b) < \ln 7$.

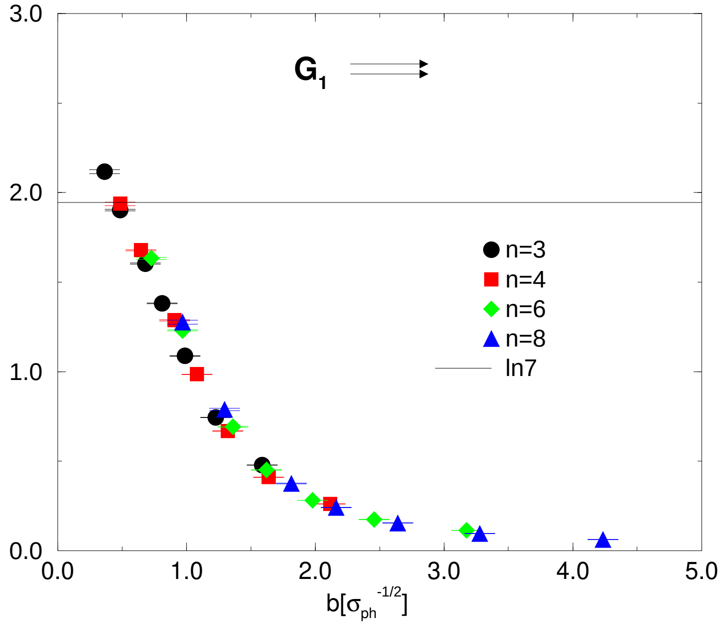

Figure 2: The self coupling versus b in the one-current case.

4. Perfect action and perfect operator for monopole current

The monopole action seems to satisfy the scaling behavior. The scaling is the characteristic of the perfect action. The perfect action reproduces the continuum rotational invariance. To test the rotational invariance, we have to determine the form of physical operators on the blocked lattice. For that purpose, as done in the SU(2) case, we perform a block-spin transformation from the small a-lattice ($a \to 0$) to the finite $b = na$ lattice analytically, restricting ourselves to a simple case of two-point monopole interactions with a monopole Wilson loop. Note that two-point interactions are dominant for large b region also in pure SU(3) .

We start from the following action on the a-lattice:

$$Z[J] = \sum_{k^{(a)} \in \mathbb{Z}} \delta_{\partial'_\mu k^{(a)}_\mu, 0} \delta_{\Sigma_a k^{(a)}, 0} \exp\Bigg\{ - \sum_{a,s,s'} k^{(a)}_\mu(s) D_0(s-s') k^{(a)}_\mu(s') + \\ + 2\pi i \sum_s N_\mu(s) k^{(1)}_\mu(s) \Bigg\}, \tag{4.1}$$

$$N_\mu(s) = \sum_{s'} \Delta^{-1}(s-s') \frac{1}{2} \epsilon_{\mu\alpha\beta\gamma} \partial_\alpha S_{\beta\gamma}(s' + \hat{\mu}) \,, \qquad \partial'_\alpha S_{\alpha\beta}(s) = J_\beta(s) \,. \tag{4.2}$$

Here $J_\beta(s)$ is an electric current around the Wilson loop. Note that in eq. (4.1), the static potential between quark and antiquark with $a = 1$ is considered. We have used the monopole Wilson loop operator as in ref. [26]. As the surface $S_{\alpha\beta}$, we can take any open surface with the fixed boundary C, since the monopole Wilson loop operator with the closed surface is unity due to 4-dimensional linking number in the continuum limit. Here we take the flat surface as $S_{\alpha\beta}$. We expand $D_0(s)$ and adopt the following first three terms for simplicity:

$$D_0(s-s') = \alpha\Delta^{-1}(s-s') + \beta\delta_{s,s'} + \gamma\Delta(s-s') \,. \tag{4.3}$$

The above form of the monopole action is derived from the dual Ginzburg-Landau (DGL) theory also (for details, see appendix).

We perform the block-spin transformation, using the definition of n-blocked monopole currents $K^{(a)}$ in eq. (2.6):

$$\begin{aligned} Z[K^{(a)}, J] = & \sum_{k^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu k_\mu^{(a)},0}\delta_{\Sigma_a k^{(a)},0} \times \\ & \times \exp\Big\{-\sum_{a,s,s'} k_\mu^{(a)}(s)D_0(s-s')k_\mu^{(a)}(s') + 2\pi i\sum_s N_\mu(s)k_\mu^{(1)}(s)\Big\} \times \\ & \times \prod_a \delta\left(K_\mu^{(a)}(s^{(n)}) - \sum_{i,j,k=0}^{n-1} k_\mu^{(a)}\left(ns^{(n)} + (n-1)\hat\mu + i\hat\nu + j\hat\rho + k\hat\sigma\right)\right) \end{aligned} \tag{4.4}$$

$$\begin{aligned} = & \int_{-\pi}^{\pi}\mathcal{D}\boldsymbol{\gamma}\int_{-\pi}^{\pi}\mathcal{D}\boldsymbol{B}\int_{-\pi}^{\pi}\mathcal{D}\phi \times \\ & \times \sum_{k^{(a)}\in\mathbb{Z}} \exp\Big\{-\sum_{a,s,s'} k_\mu^{(a)}(s)D_0(s-s')k_\mu^{(a)}(s') + 2\pi i\sum_s N_\mu(s)k_\mu^{(1)}(s) + \\ & \qquad + i\sum_{a,s} B^{(a)}(s)\partial_\mu{}' k_\mu^{(a)}(s) + i\sum_s \phi_\mu(s)\sum_a k_\mu^{(a)}(s) + \\ & \qquad + i\sum_{a,s^{(n)}} \gamma_\mu^{(a)}(s^{(n)})(K_\mu^{(a)}(s^{(n)}) - \sum_{i,j,k=0}^{n-1} k_\mu^{(a)})\Big\} , \end{aligned} \tag{4.5}$$

where we have introduced auxiliary fields ϕ, $B^{(a)}$ and $\gamma^{(a)}$ for the constraints $\sum_a k^{(a)} = 0$, $\partial'_\mu k_\mu^{(a)}(s) = 0$ and the definition of $K^{(a)}$, respectively. Here we have used such notations as $\mathcal{D}\boldsymbol{\gamma} \equiv \prod_a \mathcal{D}\gamma^{(a)}$. There are following identities for integers $m^{(a)}(s)$, $n_\mu^{(a)}(s^{(n)})$ and $P_\mu(s)$:

$$\exp\left\{2\pi i m^{(a)}(s)\partial_\mu{}' k_\mu^{(a)}(s)\right\} = 1 \,, \tag{4.6}$$

$$\exp\left\{2\pi i n_\mu^{(a)}(s^{(n)})(K_\mu^{(a)}(s^{(n)}) - \sum_{i,j,k=0}^{n-1} k_\mu^{(a)})\right\} = 1 \,, \tag{4.7}$$

$$\exp\left\{2\pi i P_\mu(s)\sum_a k_\mu^{(a)}(s)\right\} = 1 \,. \tag{4.8}$$

Then we can change the integral region of $\gamma^{(a)}$, $B^{(a)}$ and ϕ from the first Brillouin zone to the infinite region. Using the Poisson summation formula

$$\sum_{k_\mu^{(a)}\in\mathbb{Z}} f[k_\mu^{(a)}] = \text{const.}\int_{-\infty}^{\infty} dF_\mu^{(a)} \sum_{l_\mu^{(a)}\in\mathbb{Z}} \exp\left\{2\pi i F_\mu^{(a)} l_\mu^{(a)}\right\} f[F_\mu^{(a)}] \,, \tag{4.9}$$

$Z[K^{(a)}, J]$ becomes

$$Z[K^{(a)}, J] = \int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{\gamma} \int_{-\infty}^{\infty} \mathcal{D}\phi \int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{B} \int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{F} \times$$
$$\times \sum_{l^{(a)} \in \mathbb{Z}} \exp\Bigg\{ -\sum_{a,s,s'} F_\mu^{(a)}(s) D_0(s-s') F_\mu^{(a)}(s') + 2\pi i \sum_s N_\mu(s) F_\mu^{(1)}(s) +$$
$$+ i \sum_{a,s} \left(-\partial_\mu B^{(a)}(s) + \phi_\mu(s) + 2\pi l_\mu^{(a)}(s) \right) F_\mu^{(a)}(s) -$$
$$- i \sum_{a,s^{(n)}} \gamma_\mu^{(a)}(s^{(n)}) \sum_{i,j,k=0}^{n-1} F_\mu^{(a)} \Bigg\} . \qquad (4.10)$$

Writing the lattice spacing explicitly, we get

$$-ia^4 n \sum_{a,s^{(n)}} \gamma_\mu^{(a)}(nas^{(n)}) \sum_{i,j,k=0}^{n-1} F_\mu^{(a)}(nas^{(n)} + (n-1)a\hat{\mu} + ia\hat{\nu} + ja\hat{\rho} + ka\hat{\sigma}) =$$
$$= -ia^4 \sum_{a,s} X_\mu^{(a)}(as) F_\mu^{(a)}(as) \qquad (4.11)$$

and

$$X_\mu^{(a)}(as) \equiv na^4 \sum_{s^{(n)}} \gamma_\mu^{(a)}(nas^{(n)}) \delta(nas_\mu^{(n)} + (n-1)a - as_\mu) \times$$
$$\times \prod_{i(\neq\mu)} \sum_{I=0}^{n-1} \delta(nas_i^{(n)} + Ia - as_i) . \qquad (4.12)$$

Integrating out $F^{(a)}$ and $B^{(a)}$, we find

$$Z[K^{(a)}, J] = \int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{\gamma} \exp\left\{ ib^4 \sum_{a,s^{(n)}} \gamma_\mu^{(a)}(bs^{(n)}) K_\mu^{(a)}(bs^{(n)}) \right\} \int_{-\infty}^{\infty} \mathcal{D}\phi \times$$
$$\times \exp\Bigg\{ -\frac{1}{4} a^8 \sum_{s,s'} V_\mu^{(a)}(as) A_{\mu\nu}(as - as') V_\nu^{(a)}(as') \Bigg\} \times$$
$$\times \sum_{al^{(a)} \in \mathbb{Z}} \exp\Bigg\{ -\pi^2 a^8 \sum_{a,s,s'} l_\mu^{(a)}(as) A_{\mu\nu}(as - as') l_\nu^{(a)}(as') +$$
$$+ \pi^2 a^8 \sum_{a,s,s'} V_\mu^{(a)}(as) A_{\mu\nu}(as - as') l_\nu^{(a)}(as) \Bigg\} , \qquad (4.13)$$

where

$$V_\mu^{(a)}(as) \equiv X_\mu^{(a)}(as) - \phi_\mu(as) - 2\pi N_\mu(as) \delta_{a,1} ,$$
$$A_{\mu\nu}(as - as') \equiv \left\{ \delta_{\mu\nu} - \frac{\partial_\mu \partial'_\nu}{\sum_\rho \partial_\rho \partial'_\rho} \right\} D_0^{-1}(as - as') . \qquad (4.14)$$

Here we consider $l^{(a)} = 0$ alone, since we finally take the $a \to 0$ limit in the sum with respect to $al^{(a)} \in \mathbb{Z}$. We change the variables from $\gamma^{(a)}$ to $\gamma'^{(a)}$:

$$\gamma'^{(1)} \equiv \gamma^{(2)} - \gamma^{(3)}\,, \qquad \gamma'^{(2)} \equiv \gamma^{(3)} - \gamma^{(1)}\,, \qquad \gamma'^{(3)} \equiv \gamma^{(1)} - \gamma^{(2)}\,. \tag{4.15}$$

Integrating out $\gamma'^{(a)}$ and ϕ_μ, we get the monopole action in terms of $K^{(a)}$ as follows:

$$\begin{aligned} Z[J] = \exp\Bigg\{ &-\frac{2}{3}\pi^2 a^8 \sum_{s,s'} N_\mu(as) A_{\mu\nu}(as - as') N_\nu(as') + \\ &+ \frac{2}{3}\pi^2 b^8 \sum_{s^{(n)},s'^{(n)}} B_\mu(bs^{(n)}) A'^{-1}_{\mu\nu}(bs^{(n)} - bs'^{(n)}) B_\nu(bs'^{(n)}) \Bigg\} Z_{\mathrm{mon}}[J]\,, \end{aligned} \tag{4.16}$$

where

$$\begin{aligned} B_\mu(bs^{(n)}) \equiv \lim_{\substack{a\to 0\\ n\to\infty}} &\frac{1}{n^3} a^8 \sum_{s,s',\nu} \delta(nas^{(n)}_\mu + (n-1)a - as_\mu) \times \\ &\times \prod_{i(\neq\mu)} \sum_{I=0}^{n-1} \delta(nas^{(n)}_i + Ia - as_i) \left\{ \delta_{\mu\nu} - \frac{\partial_\mu \partial'_\nu}{\sum_\rho \partial_\rho \partial'_\rho} \right\} \times \\ &\times D_0^{-1}(as - as') N_\nu(as')\,, \end{aligned} \tag{4.17}$$

$$\begin{aligned} A'_{\mu\nu}(bs^{(n)} - bs'^{(n)}) \equiv \lim_{\substack{a\to 0\\ n\to\infty}} &\frac{1}{n^6} a^8 \sum_{s,s'} \delta(nas^{(n)}_\mu + (n-1)a - as_\mu) \times \\ &\times \prod_{i(\neq\mu)} \sum_{I=0}^{n-1} \delta(nas^{(n)}_i + Ia - as_i) \left\{ \delta_{\mu\nu} - \frac{\partial_\mu \partial'_\nu}{\sum_\rho \partial_\rho \partial'_\rho} \right\} \times \\ &\times D_0^{-1}(as - as') \delta(nas'^{(n)}_\nu + (n-1)a - as'_\nu) \times \\ &\times \prod_{j(\neq\nu)} \sum_{J=0}^{n-1} \delta(nas'^{(n)}_j + Ja - as'_j)\,. \end{aligned} \tag{4.18}$$

$Z_{\mathrm{mon}}[J]$ is the dynamical monopole part:

$$\begin{aligned} Z_{\mathrm{mon}}[J] = &\sum_{K^{(a)} \in \mathbb{Z}} \delta_{\partial'_\mu K^{(a)}_\mu, 0} \delta_{\sum_a K^{(a)}, 0} \times \\ &\times \exp\Bigg\{ -b^8 \sum_{a, s^{(n)}, s'^{(n)}} K^{(a)}_\mu(bs^{(n)}) A'^{-1}_{\mu\nu}(bs^{(n)} - bs'^{(n)}) K^{(a)}_\nu(bs'^{(n)}) + \\ &+ 2\pi i b^8 \sum_{s^{(n)}, s'^{(n)}} B_\mu(bs^{(n)}) A'^{-1}_{\mu\nu}(bs^{(n)} - bs'^{(n)}) K^{(1)}_\nu(bs'^{(n)}) \Bigg\}\,. \end{aligned} \tag{4.19}$$

The interactions of the perfect action on the b-lattice in eq. (4.19) depend on directions. This is consistent with the numerical data.

The spectrum of $K^{(a)}_\mu(bs^{(n)})$ is found to be equivalent to that in the continuum theory as discussed in the SU(2) case [26].

5. String representation and rotational invariance

When we transform $Z_{\mathrm{mon}}[J]$ in eq. (4.19) into the string representation as in SU(2) [31, 32], we can estimate the static potential analytically.

First we change the variables as follows:

$$K^{(1)} = j^{(2)} - j^{(3)}\,, \qquad K^{(2)} = j^{(3)} - j^{(1)}\,, \qquad K^{(3)} = j^{(1)} - j^{(2)}\,. \tag{5.1}$$

The conservation laws of monopole currents $\partial'_\mu K_\mu^{(a)} = 0$ leads us to $\partial'_\mu j_\mu^{(1)} = \partial'_\mu j_\mu^{(2)} = \partial'_\mu j_\mu^{(3)}$. The conditions are expressed as

$$\int_{-\pi}^{\pi} \mathcal{D}\boldsymbol{\varphi}\delta\left(\sum_a \varphi^{(a)}\right) \exp\left\{i\sum_{a,s} j_\mu^{(a)}(s)\partial_\mu \varphi^{(a)}(s)\right\}. \tag{5.2}$$

Then $Z_{\mathrm{mon}}[J]$ is reduced to the following:

$$\begin{aligned}
Z_{\mathrm{mon}}[J] = & \int_{-\pi}^{\pi} \mathcal{D}\boldsymbol{\varphi}\delta\left(\sum_a \varphi^{(a)}\right) \sum_{j^{(a)}\in\mathbb{Z}} \exp\Big\{-2\sum_{a,s,s'} j_\mu^{(a)}(s) {A'}^{-1}_{\mu\nu}(s-s') j_\nu^{(a)}(s') + \\
& +2\sum_{a<b,s,s'} j_\mu^{(a)}(s) {A'}^{-1}_{\mu\nu}(s-s') j_\nu^{(b)}(s') + i\sum_{a,s} j_\mu^{(a)}(s)\partial_\mu\varphi^{(a)}(s) + \\
& +2\pi i \sum_{s,s'} B_\mu(s) {A'}^{-1}_{\mu\nu}(s-s')(j_\nu^{(2)}(s') - j_\mu^{(3)}(s'))\Big\} \qquad (5.3)\\
= & \int_{-\infty}^{\infty} \mathcal{D}\vec{C} \int_{-\pi}^{\pi} \mathcal{D}\boldsymbol{\varphi}\delta\left(\sum_a \varphi^{(a)}\right) \times \\
& \times \sum_{j^{(a)}\in\mathbb{Z}} \exp\Big\{-\frac{1}{8}\sum_{s,s'} \vec{C}_\mu(s) A'_{\mu\nu}(s-s')\vec{C}_\nu(s') + \\
& + i\sum_{a,s} j_\mu^{(a)}(s)\left(\partial_\mu\varphi^{(a)}(s) + \vec{\epsilon}_a\cdot\vec{C}_\mu(s)\right) + \\
& + 2\pi i \sum_{s,s'} B_\mu(s) {A'}^{-1}_{\mu\nu}(s-s')(j_\nu^{(2)}(s') - j_\nu^{(3)}(s'))\Big\}, \qquad (5.4)
\end{aligned}$$

where auxiliary fields $\vec{C}_\mu \equiv (C_\mu^3, C_\mu^8)$ are introduced and $\vec{\epsilon}_a$ are the SU(3) root vectors: $\vec{\epsilon}_1 = (1,0)$, $\vec{\epsilon}_2 = (-1/2, -\sqrt{3}/2)$, $\vec{\epsilon}_3 = (-1/2, \sqrt{3}/2)$. Since $\sum_a \varphi^{(a)} = 0$ and $\sum_a \vec{\epsilon}_a = 0$, we get an identity

$$\exp\left\{-\frac{i}{2\pi}\sum_s \phi_\mu(s)\sum_a\left(\partial_\mu\varphi^{(a)}(s) + \vec{\epsilon}_a\cdot\vec{C}_\mu(s)\right)\right\} = 1\,. \tag{5.5}$$

The Poisson summation formula

$$\sum_{l_\mu^{(a)}\in\mathbb{Z}} \exp\left\{2\pi i\sum_a F_\mu^{(a)} l_\mu^{(a)} + i\phi_\mu \sum_a l_\mu^{(a)}\right\} = \sum_{j_\mu^{(a)}\in\mathbb{Z}} \prod_a \delta\left(F_\mu^{(a)} + \frac{\phi_\mu}{2\pi} - j_\mu^{(a)}\right) \tag{5.6}$$

gives us

$$\int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{F} \sum_{l^{(a)}\in\mathbb{Z}} \delta_{\Sigma_a l^{(a)},0} \exp\left\{2\pi i \sum_{a,s} F_\mu^{(a)}(s) l_\mu^{(a)}(s)\right\} f(F_\mu^{(a)}) =$$
$$= \text{const.} \int_{-\infty}^{\infty} \mathcal{D}\phi \sum_{j^{(a)}\in\mathbb{Z}} f\left(j_\mu^{(a)} - \frac{\phi_\mu}{2\pi}\right). \qquad (5.7)$$

Then $Z_{\text{mon}}[J]$ becomes

$$Z_{\text{mon}}[J] = \int_{-\infty}^{\infty} \mathcal{D}\boldsymbol{F} \int_{-\infty}^{\infty} \mathcal{D}\overrightarrow{C} \int_{-\pi}^{\pi} \mathcal{D}\boldsymbol{\varphi}\delta\left(\sum_a \varphi^{(a)}\right) \sum_{l^{(a)}\in\mathbb{Z}} \delta_{\Sigma_a l^{(a)},0} \times$$
$$\times \exp\Bigg\{-\frac{1}{8}\sum_{s,s'} \overrightarrow{C}_\mu(s) A'_{\mu\nu}(s-s')\overrightarrow{C}_\nu(s') +$$
$$+ i\sum_{a,s} F_\mu^{(a)}(s)\left(\partial_\mu \varphi^{(a)}(s) + \vec{\epsilon}_a \cdot \overrightarrow{C}_\mu(s) + 2\pi l_\mu^{(a)}(s)\right) +$$
$$+ 2\pi \sum_{s'} \left(F_\mu^{(2)}(s) - F_\mu^{(3)}(s)\right) A'^{-1}_{\mu\nu}(s-s') B_\nu(s')\Bigg\}. \qquad (5.8)$$

Here we perform the Berezinski-Kosterlitz-Thouless (BKT) transformation [33]:

$$l_\nu^{(a)}(s) = s_\mu^{(a)}(s) + \partial_\mu r^{(a)}(s), \qquad \partial_{[\mu} s_{\nu]}^{(a)}(s) = \sigma_{\mu\nu}^{(a)}(s),$$
$$\partial_\mu \varphi^{(a)}(s) + 2\pi l_\mu^{(a)}(s) = \partial_\mu \varphi_{nc}^{(a)}(s) - 2\pi \sum_{s'} \partial'_\nu \Delta^{-1}(s-s')\sigma_{\nu\mu}^{(a)}(s'), \qquad (5.9)$$

where $\varphi_{nc}^{(a)}(s) \equiv \varphi^{(a)}(s) - 2\pi \sum_{s'} \Delta^{-1}(s-s')\partial'_\nu s_\nu^{(a)}(s') + 2\pi r^{(a)}(s)$ is non-compact. The plaquette variable $\sigma_{\mu\nu}^{(a)}$ satisfies a conservation law $\partial_{[\alpha}\sigma_{\mu\nu]}^{(a)}(s) = 0$ and a constraint $\sum_a \sigma_{\mu\nu}^{(a)} = 0$ due to $\sum_a l_\mu^{(a)} = 0$. Using the condition $\partial'_\mu B_\mu = 0$, we integrate out $F_\mu^{(a)}$, $\overrightarrow{C}_\mu$ and $\varphi_{nc}^{(a)}$. We get the string representation:

$$Z_{str}[J] = \exp\left\{-\frac{2}{3}\pi^2 \sum_{s,s'} B_\mu(s) A'^{-1}_{\mu\nu}(s-s') B_\nu(s')\right\} \sum_{\sigma^{(a)}\in\mathbb{Z}} \delta_{\partial_{[\alpha}\sigma_{\mu\nu]}^{(a)},0}\delta_{\Sigma_a \sigma^{(a)},0} \times$$
$$\times \exp\Bigg\{-\frac{\pi^2}{3}\sum_{a,s,s'} \partial'_\alpha \sigma_{\mu\alpha}^{(a)}(s) H_{\mu\nu}(s-s')\partial'_\beta \sigma_{\nu\beta}^{(a)}(s') +$$
$$+ \frac{2\pi^2}{3}\sum_{s,s'}\left(\sigma_{\mu\nu}^{(2)}(s) - \sigma_{\mu\nu}^{(3)}(s)\right)\partial_\nu \Delta^{-1}(s-s') B_\mu(s')\Bigg\}, \qquad (5.10)$$

$$H_{\mu\nu}(s-s') = \sum_{s_1} A'_{\mu\nu}(s-s_1)\Delta^{-2}(s_1-s'). \qquad (5.11)$$

Since the numerical results show that the couplings of the monopole action are weak for large b region, we can use the strong coupling expansion in the dual string model. Then we see that the quantum fluctuations can be neglected as in SU(2) case [26]. The classical part in eq. (5.10) cancels the second classical term in eq. (4.17). As a result, the classical part of the expectation value of the Wilson loop is reduced to

$$\langle W(C)\rangle_{\rm cl} = \exp\left\{-\frac{2}{3}\pi^2\int_{-\infty}^{\infty} d^4x d^4y N_\mu(x) A_{\mu\nu}(x-y) N_\nu(y)\right\}. \tag{5.12}$$

Since the classical part is written in the continuum form, the continuum rotational invariance is trivial. The static potential $V(Ib,0,0)$ and $V(Ib,Ib,0)$ can be evaluated as in SU(2) . We take the following plaquette sources as $S_{\alpha\beta}(x)$ for $V(Ib,0,0)$:

$$S_{\alpha<\beta}(x) = \delta_{\alpha 1}\delta_{\beta 4}\theta(x_1)\theta(Ib-x_1)\theta(x_4)\theta(Tb-x_4)\delta(x_2)\delta(x_3) \tag{5.13}$$

and for $V(Ib,Ib,0)$:

$$\begin{aligned} S_{\alpha<\beta}(x) = {} & (\delta_{\alpha 1}\delta_{\beta 4}+\delta_{\alpha 2}\delta_{\beta 4})\delta(x_3)\theta(x_4)\theta(Tb-x_4)\times \\ & \times\theta(x_1)\theta(Ib-x_1)\theta(x_2)\theta(Ib-x_2)\delta(x_1-x_2)\,, \end{aligned} \tag{5.14}$$

respectively. Using the following formula

$$\lim_{T\to\infty}\left(\frac{\sin\alpha T}{\alpha}\right)^2 = \pi T\delta(\alpha)\,, \tag{5.15}$$

we get the static potentials

$$\begin{aligned} V(Ib,0,0) &= -\lim_{T\to\infty}\frac{1}{Tb}\ln\langle W(Ib,0,0,Tb)\rangle_{\rm cl} \\ &\xrightarrow[I\to\infty]{} \frac{2\pi^2}{3}(Ib)^2\int\frac{d^2p}{2\pi^2}\left[\frac{1}{\Delta D_0}\right](0,p_2,p_3,0) \\ &= \frac{\pi\kappa Ib}{3}\ln\frac{m_1}{m_2}\,, \end{aligned} \tag{5.16}$$

$$V(Ib,Ib,0) = \frac{\sqrt{2}\pi\kappa Ib}{3}\ln\frac{m_1}{m_2}\,, \tag{5.17}$$

where $\kappa(m_1^2-m_2^2)=\gamma^{-1}$, $m_1^2+m_2^2=\beta/\gamma$, $m_1^2m_2^2=\alpha/\gamma$. The static potential has the linear term alone and the rotational invariance is recovered completely. The string tension is evaluated as

$$\sigma_{\rm cl} = \frac{\pi\kappa}{3}\ln\frac{m_1}{m_2}\,. \tag{5.18}$$

This is consistent with the results [34]. The m_1^{-1} and the m_2^{-1} could be regarded as the coherence and the penetration lengths in type-2 superconductor.

6. Estimate of the string tension

Ths string tension in eq. (5.18) is written by the parameters of the monopole action taken on the a-lattice. The parameters can be determined by comparing the theoretical perfect action in eq. (4.19) with the numerical results of the monopole action on the b-lattice. $A'^{-1}_{\mu\nu}$ in eq. (4.18) fixed the gauge is calculated in ref. [26]. The results of the comparison are seen in figure 3. The agreement is rather good. The value of the string tension is $\sqrt{\sigma_{\mathrm{cl}}/\sigma_{ph}} \sim 1.22$ for large b as in figure 4. The result is not so bad, although the discrepancy is not negligible.

In order to study whether the difference is due to the ambiguity of the fit of D_0, we try to estimate the string tension without determining the parameters on the a-lattice. The following monopole Wilson loop on the b-lattice is considered:

$$\langle W(C)\rangle = \frac{1}{Z}\sum_{K^{(a)}\in\mathbb{Z}} \delta_{\partial'_\mu K^{(a)}_\mu,0}\delta_{\Sigma_a K^{(a)},0} \times \\ \times \exp\left\{-\sum_{a,s,s'} K^{(a)}_\mu(s) D^{-1}_{\mu\nu}(s-s')K^{(a)}_\nu(s') + 2\pi i\sum_s N_\mu(s)K^1_\mu(s)\right\}, \tag{6.1}$$

where the naive monopole Wilson loop operator (A.15) on the coarse b-lattice is used and $D^{-1}_{\mu\nu}$ corresponds to $A'^{-1}_{\mu\nu}$. Transforming eq. (6.1) to the string representation and neglecting the quantum fluctuations as in the previous section, we get the classical part of the Wilson loop as follows:

$$\langle W(C)\rangle_{\mathrm{cl}} = \exp\left\{-\frac{2}{3}\pi^2\sum_{s,s'} N_\mu(s)D_{\mu\nu}(s-s')N_\nu(s')\right\}. \tag{6.2}$$

It is found theoretically that the static potential $V(Ib,0,0)$ evaluated by eq. (6.2) agrees with eq. (5.16) in the $I\to\infty$ case. Using eq. (6.2), the string tension becomes

$$\sigma_{\mathrm{cl}} = \frac{\pi^2}{6}\int_{-\pi}^{\pi}\frac{d^2k}{(2\pi)^2}\frac{1}{(\sin^2\frac{k_2}{2}+\sin^2\frac{k_3}{2})^2} \times \\ \times\left[\sin^2\frac{k_3}{2}D(0,k_2,k_3,0;\hat{2}) + \sin^2\frac{k_2}{2}D(0,k_2,k_3,0;\hat{3})\right], \tag{6.3}$$

$$D_{\mu\nu}(s-s') = \delta_{\mu\nu}\int_{-\pi}^{\pi}\frac{dk^4}{(2\pi)^4}D(k_1,k_2,k_3,k_4;\hat{\mu})e^{ik(s-s')}. \tag{6.4}$$

We estimate the string tension from eq. (6.3) with the numerical data of the monopole action on the b-lattice.

The results are seen in figure 4. The results are not so different from those of the first method for large b. The b-independence holds roughly for $b > 2$ and $\sqrt{\sigma_{\mathrm{cl}}/\sigma_{ph}} \sim 1.27$ for large b. The difference from the physical value is also seen in this method.

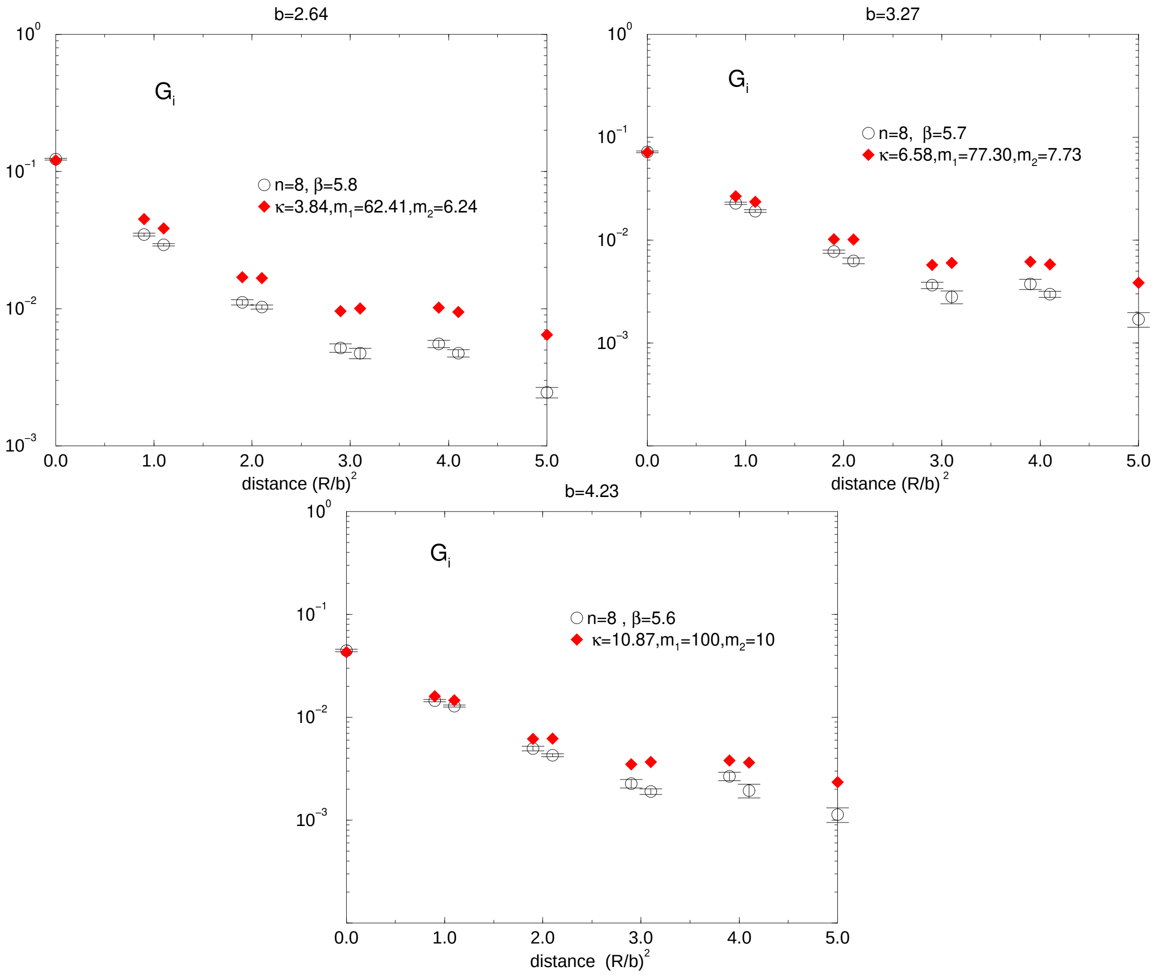

Figure 3: The coupling G_i versus distance. The data from (i) perfect action (diamonds) and (ii) Swendsen method (circles). The values of dimensionless κ, m_1 and m_2 are shown.

Using the same method as in SU(2) [26], the effect of the quantum fluctuation can be evaluated.

The order of the correction for the string tension becomes

$$-\frac{8}{b^2}\exp\{-5\Pi(0)b^2\}\,, \tag{6.5}$$

where $\Pi(0)$ represents the coupling of the self-interaction in the string model. For example, the value of eq. (6.5) is -2.6×10^{-10} for $b = 3.27$. The correction is very small in large b region.

In order to clarify the origin of the difference, we have to study the effects of systematic errors carefully.

As a systematic error, the difference between D-T monopoles used in the simulation and the monopoles in the analytical calculations may be important. The magnetic charge of the latter ranges from $-\infty$ to ∞, while D-T monopoles take only the restricted values of magnetic charges. This is under investigation.

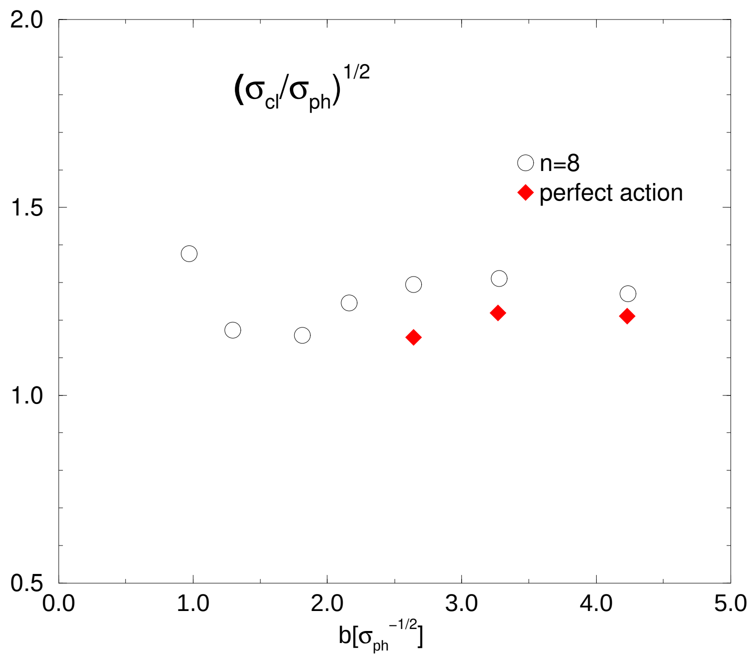

Figure 4: The string tension versus b. The data from (i) eq. (5.18) (diamonds) and (ii) eq. (6.3) (circles).

The glueball spectrum can be evaluated similarly as in SU(2) QCD [26]. The lightest 0^{++} glueball mass is determined as $M_{0^{++}} = 2m_2$. For $b = 4.23$, $2m_2/\sqrt{\sigma_{ph}} = 3.90$. The result is not so different from 3.64 ± 0.11 [35].

7. Conclusions

We have studied an effective monopole action in pure SU(3) lattice QCD. The following results are obtained:

1. We have found that the SU(3) monopole action can be derived clearly for various n-blocked monopoles in MA gauge numerically, using the extended Swendsen method in the two-current and the one-current cases. We perform $n = 2 \sim 8$ step block-spin transformations on the 48^4 lattice. In the two-current case, we obtain an almost perfect lattice monopole action for the infrared region of SU(3) QCD, since the action seems to depend on a physical scale b alone. The simple 4, 6-point interactions become negligibly small as compared with two-point interactions for large b region as in SU(2) . Thus two-point monopole interactions are dominant in the infrared region. In the one-current case, if the physical scale b is taken in unit of the string tension $\sqrt{\sigma_{ph}}$, the scaling which was not seen in the previous study [27] holds beautifully. Monopole condensation occurs in the large b region from the energy-entropy balance.

2. The perfect action satisfies the continuum rotational invariance. When we try to study the restoration of the rotational invariance of the static potential, we have to know the correct form of the perfect operator. For that purpose, a block-spin transformation from the small a-lattice ($a \to 0$) to the finite $b = na(\beta)$ lattice is performed analytically in a simple restricted case of a quadratic monopole action as in SU(2) case. The perfect monopole action and the perfect operator evaluating the static potential between electric charges on the b-lattice are derived. The quadratic SU(3) monopole action is simple, but it corresponds to the London limit of the DGL theory which is non-trivially interacting field theory.

3. The SU(3) monopole action can be transformed into the string model. There are three strings $\sigma^{(a)}$ satisfying one constraint $\sum_a \sigma^{(a)} = 0$. This is consistent with the results in the continuum limit [36, 37]. Since the monopole interactions are weak for large b region, the dual string interactions are strong. Using the strong coupling expansion, the static potential in the long distance is calculated by the classical part alone analytically. The static potential has a linear term alone and the restoration of the rotational invariance can be seen explicitly. The string tension is estimated from the numerical data of the monopole action. The result is rather good, but the difference from the physical string tension is not negligible. The same thing happens in SU(2) case [26]. It seems that there still exist some systematic errors.

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A. Monopole action from the DGL theory

In SU(2), it is already known that the monopole action can be derived from the dual abelian higgs model on the fine a-lattice [8, 32, 38]. We show how to derive the monopole action from the DGL theory on the lattice in SU(3). In DGL, $\vec{A}_\mu \equiv (A^3_\mu, A^8_\mu)$ are used as $\mathrm{U}(1)^2$ photon fields, where $A_\mu = A^i_\mu \lambda^i/2$ (λ^i are Gell-Mann matrices). The magnetic charges $\vec{m}$ are distributed on the SU(3) root lattice: $\vec{m} = g_m \sum_{a=1}^{3} \xi_a \vec{\epsilon}_a$, $\xi_a \in \mathbb{Z}$, $g_m = 4\pi/g$ (g is the SU(3) coupling constant). The DGL lagrangean in the continuum limit [34] is

$$\mathcal{L}_{DGL} = \frac{1}{4}\left(\partial_\mu \overrightarrow{C}_\mu - \partial_\nu \overrightarrow{C}_\mu\right)^2 + \sum_{a=1}^{3}\left(\left|\left(\partial_\mu + ig_m\vec{\epsilon}_a\cdot\overrightarrow{C}_\mu\right)\phi^{(a)}\right|^2 + \lambda(|\phi^{(a)}|^2 - v^2)^2\right), \tag{A.1}$$

where $\overrightarrow{C}_\mu \equiv (C_\mu^3, C_\mu^8)$ are dual $\mathrm{U}(1)^2$ gauge fields and $\phi^{(a)} = \rho^{(a)}\exp(i\varphi^{(a)})$ are monopole fields with a constraint $\sum_a \varphi^{(a)} = 0$.

In the below, we use the notations of differential forms on the lattice [39]. The lattice DGL action [40] becomes

$$\begin{aligned} S_{DGL}[\overrightarrow{C}, \phi^{(a)}] = {} & \frac{1}{2g_m^2}||d\overrightarrow{C}||^2 - \gamma\sum_{a,s,\mu}\left(\phi_s^{(a)*}U_\mu^{(a)}(s)\phi_{s+\hat{\mu}}^{(a)} + \text{h.c.}\right) + \\ & +\lambda\sum_{a,s}(\phi_s^{(a)*}\phi_s^{(a)} - 1)^2 + \sum_{a,s}\phi_s^{(a)*}\phi_s^{(a)}, \end{aligned} \tag{A.2}$$

where $U^{(a)} = \exp(i\vec{\epsilon}_a\cdot\overrightarrow{C})$. We modify the partition function of the DGL model using the Villain approximation $e^{\alpha\cos\psi} \to \sum_{l\in\mathbb{Z}} e^{-\frac{\alpha}{2}(\psi+2\pi l)^2}$. In DGL, the summations of $l^{(a)}$ appear due to 2π periodicity of $\cos(d\varphi^{(a)} + \vec{\epsilon}_a\cdot\overrightarrow{C})$. Then there is a constraint $\sum_a l^{(a)} = 0$, since $\sum_a(d\varphi^{(a)} + \vec{\epsilon}_a\cdot\overrightarrow{C}) = 0$. The partition function of the DGL theory is given by

$$\begin{aligned} Z_{DGL} = {} & \int_{-\infty}^{\infty}\mathcal{D}\overrightarrow{C}\int_{-\pi}^{\pi}\mathcal{D}\boldsymbol{\varphi}\delta\left(\sum_a\varphi^{(a)}\right)\int_0^{\infty}\mathcal{D}\boldsymbol{\rho}^{-2}\sum_{l^{(a)}\in Z}\delta_{\sum_a l^{(a)},0}\times \\ & \times\exp\Big\{-\frac{1}{2g_m^2}||d\overrightarrow{C}||^2 - \gamma\sum_{a,s,\mu}\rho_s^{(a)}\rho_{s+\hat{\mu}}^{(a)}\left(d\varphi^{(a)} + \vec{\epsilon}_a\cdot\overrightarrow{C} + 2\pi l^{(a)}\right)_{s,\mu}^2 - \\ & -\lambda\sum_{a,s}\left((\rho_s^{(a)})^2 - 1\right)^2 - \sum_{a,s}(\rho_s^{(a)})^2\Big\}. \end{aligned} \tag{A.3}$$

Inserting

$$\begin{aligned} 1 = {} & \Big\{\prod_{a,s,\mu}(4\gamma\rho_s^{(a)}\rho_{s+\hat{\mu}}^{(a)})^{-1/2}\Big\}\int_{-\infty}^{\infty}\mathcal{D}\boldsymbol{F}\exp\Big\{-\sum_{a,s,\mu}\frac{1}{4\gamma\rho_s^{(a)}\rho_{s+\hat{\mu}}^{(a)}}\times \\ & \times\left(F_\mu^{(a)}(s) - 2i\gamma\rho_s^{(a)}\rho_{s+\hat{\mu}}^{(a)}(d\varphi^{(a)} + \vec{\epsilon}_a\cdot\overrightarrow{C} + 2\pi l^{(a)})_{s,\mu}\right)^2\Big\} \end{aligned} \tag{A.4}$$

and using the relation in eq. (5.7), we integrate out $\phi, \varphi^{(a)}$ and $\overrightarrow{C}$. The partition function is written by the variables $j^{(a)}$ in the r.h.s. of eq. (5.7) and $\rho^{(a)}$. From the integrals of $\varphi^{(a)}$, the relations $\delta j^{(1)} = \delta j^{(2)} = \delta j^{(3)}$ appear. Here monopole currents are introduced as follows:

$$k^{(1)} \equiv j^{(2)} - j^{(3)}, \qquad k^{(2)} \equiv j^{(3)} - j^{(1)}, \qquad k^{(3)} \equiv j^{(1)} - j^{(2)}. \tag{A.5}$$

They satisfy the conservation law $\delta k^{(a)} = 0$ and the constraint $\sum_a k^{(a)} = 0$. The partition function can be rewritten as follows:

$$Z_{\rm mon}[k^{(a)}] = \sum_{k^{(a)} \in \mathbb{Z}} \delta_{\delta k^{(a)},0} \delta_{\sum_a k^{(a)},0} \exp\{-S^G_{\rm mon}[k^{(a)}] - S^H_{\rm mon}[k^{(a)}]\}\,, \tag{A.6}$$

$$S^G_{\rm mon}[k^{(a)}] = \frac{g_m^2}{4} \sum_{a=1}^{3} (k^{(a)}, \Delta^{-1} k^{(a)})\,, \tag{A.7}$$

$$\begin{aligned} S^H_{\rm mon}[k^{(a)}] = &- \ln\Bigg\{ \int_0^\infty \mathcal{D}\boldsymbol{\rho}^{-2} \exp\Bigg\{ -\frac{1}{4\gamma} \sum_{s,\mu} \frac{\sum_a \rho_s^{(a)} \rho_{s+\hat\mu}^{(a)} (k_\mu^{(a)}(s))^2}{\sum_{a<b} \rho_s^{(a)} \rho_{s+\hat\mu}^{(b)}} - \\ &-\lambda \sum_{a,s} ((\rho_s^{(a)})^2 - 1)^2 - \sum_{a,s} (\rho_s^{(a)})^2 \Bigg\}\Bigg\}\,. \end{aligned} \tag{A.8}$$

The integration of $\rho^{(a)}$ in eq. (A.8) can't be performed exactly. In the London limit ($\lambda \to \infty$), $\rho^{(a)}$ are fixed to unity. $S^H_{\rm mon}[k^{(a)}]$ becomes the quadratic self interaction:

$$S^H_{\rm mon}[k^{(a)}] = \frac{1}{12\gamma} \sum_a ||k^{(a)}||^2\,. \tag{A.9}$$

The monopole action in eq. (4.1)

$$S_{\rm mon}[k^{(a)}] = \sum_a (k^{(a)}, D_0 k^{(a)})\,, \qquad D_0 = \alpha\Delta^{-1} + \beta + \gamma\Delta \tag{A.10}$$

corresponds to the modified London limit of DGL:

$$\begin{aligned} S_{DGL}[\vec{C}, \varphi^{(a)}] &= \frac{1}{8\alpha} ||d\vec{C}||^2 + \frac{1}{12} \sum_a \left(X^{(a)}, \{\beta + \gamma\Delta\}^{-1} X^{(a)} \right)\,, \\ X^{(a)} &= d\varphi^{(a)} + \vec{\epsilon}_a \cdot \vec{C} + 2\pi l^{(a)}\,. \end{aligned} \tag{A.11}$$

The Wilson loop is also calculated similarly. For simplicity, we consider the case of the London limit. Since the DGL theory is the dual theory for a color electric $\mathrm{U}(1)^2$ charge, we consider the 't Hooft loop:

$$\begin{aligned} \langle H_c(C) \rangle = &\frac{1}{Z_{DGL}} \int_{-\infty}^{\infty} \mathcal{D}\vec{C} \int_{-\pi}^{\pi} \mathcal{D}\boldsymbol{\varphi} \delta\left(\sum_a \varphi^{(a)} \right) \sum_{l^{(a)} \in \mathbb{Z}} \delta_{\sum_a l^{(a)},0} \times \\ &\times \exp\left\{ -\frac{1}{2g_m^2} \left\| d\vec{C} - 4\pi \vec{Q}_c {}^*S \right\|^2 - \gamma \sum_a \left\| d\varphi^{(a)} + \vec{\epsilon}_a \cdot \vec{C} + 2\pi l^{(a)} \right\|^2 \right\}, \end{aligned} \tag{A.12}$$

where S is a surface with the fixed boundary C: $\delta S = j$, where j is the unit current on the loop C. $\vec{Q}_c$ are color electric $\mathrm{U}(1)^2$ charges of quarks with colors c ($c = R, G, B$):

$$\vec{Q}_c = (Q_c^3, Q_c^8) = \left\{ \left(\frac{1}{2}, \frac{1}{2\sqrt{3}} \right), \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}} \right), \left(0, -\frac{1}{\sqrt{3}} \right) \right\}. \tag{A.13}$$

For $c = R$, we get

$$\langle H_R(C)\rangle = \frac{1}{Z_{\rm mon}} \sum_{k^{(a)}\in\mathbb{Z}} \delta_{\delta k^{(a)},0}\delta_{\Sigma_a k^{(a)},0} \exp\Big\{ -\frac{g_m^2}{4}\sum_a (k^{(a)}, \Delta^{-1}k^{(a)}) - \\ -\frac{1}{12\gamma}\sum_a ||k^{(a)}||^2 + 2\pi i({}^*S, \Delta^{-1}dk^{(3)}) - \frac{8\pi^2}{3g_m^2}({}^*j, \Delta^{-1*}j)\Big\}. \quad (A.14)$$

The monopole Wilson loop operators are

$$e^{2\pi i({}^*S,\Delta^{-1}dk^{(3)})}, \qquad e^{2\pi i({}^*S,\Delta^{-1}dk^{(2)})}, \qquad e^{2\pi i({}^*S,\Delta^{-1}dk^{(1)})}, \quad (A.15)$$

for $c = R, G, B$, respectively. When ${}^*S$ is a closed surface, $({}^*S, \Delta^{-1}dk^{(a)})$ in eq. (A.15) is the 4-dimensional linking number between the closed surface ${}^*S$ and the closed loop $k^{(a)}$. Hence the monopole Wilson loop is independent of the choice of the surface S. This is very important.

Let us consider the string representation of eq. (A.12). Introducing the following $\mathrm{U}(1)^2$ electric charges:

$$\boldsymbol{s}_c = (s_c^{(1)}, s_c^{(2)}, s_c^{(3)}) = \{(1,-1,0), (-1,0,1), (0,1,-1)\} \quad (A.16)$$

for $c = R, G, B$ respectively [36], the string representation can be derived as in section 5:

$$\langle H_c(C)\rangle = \frac{1}{Z_{str}} \sum_{\substack{\sigma_c^{(a)}\in\mathbb{Z}, \\ \delta^*\sigma_c^{(a)}=j_c^{(a)}}} \delta_{\Sigma_a\sigma_c^{(a)},0} \exp\Big\{ -4\pi^2\gamma\Big\{\sum_a (\sigma_c^{(a)}, (\Delta+m^2)^{-1}\sigma_c^{(a)}) + \quad (A.17) \\ + \frac{1}{m^2}\sum_a (j_c^{(a)}, (\Delta+m^2)^{-1}j_c^{(a)})\Big\}\Big\},$$

where $j_c^{(a)} \equiv s_c^{(a)}j$ are electric $\mathrm{U}(1)^2$ currents and $m^2 = 3\gamma g_m^2$ is a mass of a dual gauge field in DGL. The two-forms $\sigma_c^{(a)} \equiv s_c^{(a)}S + \sigma^{(a)}$ satisfy $\delta^*\sigma_c^{(a)} = j_c^{(a)}$ with the constraint $\sum_a \sigma_c^{(a)} = 0$. Thus we obtain the hadronic string model in the SU(3) lattice QCD.

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